PHP 2510 (Fall, 2009)

First Midterm Exam

October 22

Name: ______________________________

Solutions

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There are three questions on this exam. Show your reasoning for each answer, and circle your final answer to avoid any ambiguities.

1. Consider a family with a mother, father, and two children. Let event
   $A_1 = \{\text{mother has influenza}\}$,
   $A_2 = \{\text{father has influenza}\}$,
   $A_3 = \{\text{first child has influenza}\}$,
   $A_4 = \{\text{second child has influenza}\}$,
   $B = \{\text{at least one child has influenza}\}$,
   $C = \{\text{at least one parent has influenza}\}$,
   $D = \{\text{at least one person has influenza}\}$.

   Answer the following questions:

   (a) What does $A_1 \cap A_2$ mean?

   (b) Are A_3 and A_4 mutually exclusive? (Just answer yes or no).

   (c) Express D in terms of B and C.

   Suppose an influenza epidemic strikes the US. In 10% of such families the mother has influenza; in 10% of such families the father has influenza; and in 2% both the mother and father have influenza.

   (d) Are events A_1 and A_2 independent? Justify your answer.

   (e) What is the probability that the father has influenza given that the mother has influenza?

a) Both mother and father have influenza.

b) No.

c) $D = B \cup C$.

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d) $P(A_1) = 0.1$

$P(A_2) = 0.1$

$P(A_1 \cap A_2) = 0.02$

A_1 and A_2 are not independent because $P(A_1 \cap A_2) \neq P(A_1) \times P(A_2)$.

e) $P(A_2 | A_1)$

$$= \frac{P(A_1 \cap A_2)}{P(A_1)}$$

$$= \frac{0.02}{0.1}$$

$$= 0.2$$

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2. A study followed 60 male patients with small cell lung carcinoma (SCLC) for 5 years, and found that only 2 patients survived. Suppose that the nationwide average 5-yr survival rate of patients was 8%. Answer the following question.

   (a) What is the expected number of alive patients in the study if the nationwide survival rate holds true for the study population?

   (b) Compute $\binom{60}{2}$.

   (c) What is the exactly probability of having 2 alive patients if the survival rate is 8%?

   (d) Is there evidence of an excessive mortality in the study? That is, what is the exact probability of seeing two or less alive patients if the nationwide rate holds true?

   (e) Answer (d) using Poisson approximation.

a) $60 \times 0.8 = 4.8$ patients

b) $\binom{60}{2} = \frac{60 \times 59}{1 \times 2} = 1770$

c) Let x denote the number of patients alive. X has a binomial distribution.

$X \sim \text{Binomial}(p)$

where $p = 0.08$

$Pr(X=2) = \binom{60}{2} p^2 (1-p)^{60-2}$

$= 1770 \times 0.0064 \times 0.00794$

$= 0.09$

d) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \binom{60}{0} 0.08^0 \times 0.92^{60} + \binom{60}{1} 0.08^1 \times 0.92^{59}$

$+ \binom{60}{2} 0.08^2 \times 0.92^{58}$

$= 0.007 + 0.03 + 0.09$

$= 0.13$

e) $\lambda = np = 60 \times 0.08 = 4.8$

Using Poisson approximation,

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{e^{-4.8} \cdot 4.8^0}{0!} + \frac{e^{-4.8}\, 4.8^1}{1!} + \frac{e^{-4.8} \cdot 4.8^2}{2!}$

$= 0.008 + 0.038 + 0.095$

$= 0.14$

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3. Suppose that the triceps skin-fold thickness of men with chronic airflow limitation (CAL) can be regarded as being normally distributed with a mean of 0.95 with a standard deviation of 0.4.

   (a) What is the variance of triceps skin-fold thickness of men with CAL?

   (b) For a randomly selected man with CAL, what is the probability that his triceps skin-fold thickness is greater than 1.35 (average triceps skin-fold thickness of normal men)?

   Define X_1 and X_2 to be the triceps skin-fold thickness of two randomly-selected men with CAL. Suppose they are independent.

   (c) Find the expected value and the variance of $\frac{X_1+X_2}{2}$.

   (d) Find the expected value and the variance of $X_1 - X_2$.

   Define T_i as being whether the triceps skin-fold thickness of a randomly-selected man with CAL is greater than 1.35, i.e. $T_i = 1$ if $X_i > 1.35$, for $i = 1, 2$.

   (~~d~~ e) Find the expected value and the variance of $\frac{T_1+T_2}{2}$.

a) $\sigma^2 = 0.4^2 = 0.16$

b) $Z = \dfrac{1.35-0.95}{0.4} = 1$

$Pr(\text{tricep thickness} > 1.35)$

$= Pr(Z > 1) = 0.16$

c) X_1 and X_2 are independent

$E(X_1) = E(X_2) = 0.95$

$Var(X_1) = Var(X_2) = 0.16$

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$$E\left(\frac{X_1+X_2}{2}\right) = \frac{1}{2}\{E(X_1)+E(X_2)\} = 0.95$$

$$Var\left(\frac{X_1+X_2}{2}\right) = \frac{1}{4}Var(X_1) + \frac{1}{4}Var(X_2) = 0.08$$

d) $E(X_1 - X_2) = E(X_1) - E(X_2) = 0$

$$Var(X_1 - X_2) = 1^2 Var(X_1) + (-1)^2 Var(X_2)$$
$$= 0.16 + 0.16 = 0.32$$

e) T_i has a binomial distribution

$T_i \sim Binomial(p)$

where $p = Pr(X_i > 1.35) = 0.16$

so, $E(T_i) = p = 0.16$

$Var(T_i) = p(1-p) = 0.16 \times 0.84 = 0.134$

$$E\left(\frac{T_1+T_2}{2}\right) = \frac{E(T_1)+E(T_2)}{2} = 0.16$$

$$Var\left(\frac{T_1+T_2}{2}\right) = \frac{Var(T_1)+Var(T_2)}{4} = 0.067$$